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Methodology

Counterfactuals

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In the report **The housebuilding crisis: The UK's 4 million missing homes**, housing outcomes are defined as the number of homes per (thousand) people. This is a simple metric for general comparisons in housing quality and affordability.

One of the goals of the report was to calculate exactly how much underbuilding had taken place in the UK since 1955 compared to other countries, and how this affected outcomes in the UK.

Comparisons that simply applied the housebuilding rates of other countries to the UK generated unbuilt backlogs that were too large, as other countries in 1955 had worse outcomes and higher population growth, and therefore needed to build more.

More reasonable estimates could be produced by calculating the housebuilding required to reach the number of homes per person as other European countries in 2015 produced, but these were still unsatisfactory, for two reasons:

First, investigating differences in housebuilding by tenure was not possible with this method, despite being a key aspect of the report.

And second, inconsistencies in the data between countries make it difficult to draw comparisons based on the number of dwellings recorded in 2015. Both the UK, and wherever possible the UN data, record the total housing stock until 2000. However, the statistical agencies of many other countries today only record primary residences, and omit second homes and vacant stock. Therefore, using the publicly available 2015 dwellings per person ratios from national statistics agencies to estimate UK underbuilding from 1955 generates underestimates and inconsistent results between countries.

To solve this, Centre for Cities used a new method that combined both approaches. The report's estimates of the total amount of underbuilding in the UK from 1955 to 2015 relative to other countries use the difference between the number of homes per person that the UK would have if Britain had built at the rate of another country. This is controlling for the difference in the size of initial stock of homes per person and different population growth rates.

The underlying formula required to produce estimates of the change in the number of homes per person every year (and therefore from 1955 to 2015) is:

$$\frac{Previous\ Year's\ Housing\ Stock + Annual\ Net\ Additions}{Annual\ Change\ in\ Population}$$

With:

$$Annual\ Net\ Additions = Annual\ Houses\ Built - Annual\ Houses\ Demolished$$

This formula is used twice to create the estimate of underbuilding – first, to create the estimate of actual British net additions every year, and then again to produce an estimate of net additions for each "counterfactual" version of the UK which is adjusted for each country comparison (e.g. the Netherlands). The difference in final values between the actual British and each country's counterfactual British values that this produces is the estimate of the amount of housebuilding that would have taken place in the UK had it followed the housing policy approach of each country.

The first part of this methodology explains how the formula is calculated, which is applied without adjustments to the British data to create estimates on how many homes were added by the private and public sectors from 1955 to 2015. The second part explains how the formula is then adjusted for each counterfactual.

Part I – Calculating the actual British values

Four steps are required to calculate the formula:

1. The housing stock for each year from 1955 to 2015.
2. The net change in housing stock between 1955 and 2015.
3. The gross total number of new homes built and demolished by tenure from 1955 to 2015.
4. The net change in housing stock by tenure from 1955 to 2015.

The end estimates will not be identical to the values defined as national statistics, but the approach is being used to maintain consistency between the method used to generate the baseline UK values and the counterfactuals. The discrepancies between the official estimates are small and within reasonable margins of error (~2.3 per cent difference with the official statistic for the end 2015 estimate, for example).

Stage 1: The housing stock for each year from 1955 to 2015

Written in mathematical notation, the British value where the initial UK housing stock in 1955 is H_0. In each year t after 1955, the British building and demolition rate are b_t and d_t respectively. This produces the estimates for the years 1955, 1956, and 1957:

$$H_0$$

$$H_0 + H_0 \times (b_1 - d_1)$$

$$(H_0 + H_0 \times (b_1 - d_1)) + (H_0 + H_0 \times (b_1 - d_1)) \times (b_2 - d_2)$$

This expression can be factorized:

$$H_0$$

$$H_0(1 + (b_1 - d_1))$$

$$H_0(1 + (b_1 - d_1)) + H_0(1 + (b_1 - d_1)) \times (b_2 - d_2)$$

In the case of the UK,

$$H_0 = 15.4\ million\ , b_1 = 1.8\,\% , d_1 = 0.36\%, b_2 = 1.6\%, d_2 = 0.41\%,$$

Which gives values of $15.4\ million$, $15.7\ million$, $15.9\ million$ for the beginnings of years 1955, 1956 and 1957 respectively. This process is continued

until 2015, and allows us to generate estimates for the number of homes per person every year.

Stage 2: The net change in housing stock between 1955 and 2015

The net change in housing stock from 1955 to 2015 can be calculated as the difference between the initial year (1955) and the final year (2015). Mathematically:

$$H_{Final} = H_0 + H_0(b_1 - d_1) + (H_0 + H_0 \times (b_1 - d_1)) \times (b_2 - d_2) \ldots$$

$$H_{Change} = H_0 + H_0(b_1 - d_1) + (H_0 + H_0 \times (b_1 - d_1)) \times (b_2 - d_2) \ldots - H_0$$

When calculating the actual estimate for the UK, the values of H_0 cancel out leaving a value of:

$$H_{Change} = H_0(b_1 - d_1) + (H_0 + H_0 \times (b_1 - d_1))(b_2 - d_2)$$

In the case of the UK, the values for the final amount of housing in 2015 and the net change in stock from 1955 to 2015 are:

$$H_{Final} = 27.7\ million$$

$$H_{Change} = 12.3\ million$$

Stage 3: Calculation of gross housebuilding and gross demolitions

The net change in total dwellings is not enough if we want to generate estimates for additions by tenure. Gross housebuilding and gross demolitions are required for this, and summed together they will equal the net change in dwellings.

Total gross housebuilding in the UK is the build rate in each year multiplied by the housing stock in each year from 1955 to 2015, represented mathematically:

$$H_0 b_1 + (H_0 + H_0 \times (b_1 - d_1))\, b_2 \ldots$$

The gross number of new homes that are built differs from the net change in the number of homes, as demolitions are omitted from the gross. The gross number of demolitions from 1955 to 2015 takes the similar equation:

$$H_0 d_1 + \big(H_0 + H_0 \times (b_1 - d_1)\big) d_2 \ldots$$

As the net change in stock must equal the total gross housebuilding minus the total gross demolitions, the results can be checked according to this formula:

$$\sum H_0 b_1 + (H_0 + H_0 \times (b_1 - d_1))\, b_2 \ldots - \sum H_0 d_1 + \big(H_0 + H_0 \times (b_1 - d_1)\big) d_2 \ldots = H_{Change}$$

For the UK:

$$\sum H_0 b_1 + (H_0 + H_0 \times (b_1 - d_1))\, b_2 \ldots = 15.5\ million$$
$$\sum H_0 d_1 + \big(H_0 + H_0 \times (b_1 - d_1)\big) d_2 \ldots = 3.2\ million$$

Therefore H_{Change} = 15.5 million – 3.2 million = 12.3 million, matching the results in Stage 2.

Stage 4: Calculation of net change by tenure

However, to split net change in stock by tenure requires further assumptions for demolitions (and population growth for the counterfactuals). We have assumed that demolitions (and population growth) were allocated each year to **the same tenure proportion as the houses that were built in each year** and adjusted the net additions by tenure accordingly.

British data is available from 1955 to 2015 for gross housebuilding by tenure and gross demolitions, but not the necessary data to measure demolitions by tenure. Although for the latter part of the period some data does exist on demolitions by tenure of final occupier for England, data does not exist for data on demolitions by tenure of *construction*. As this report is concerned with the contribution of the private and public sector to total net additions of stock, it is the latter we are interested in.

To take an example - consider a local authority which built ten council houses, and four of them were sold to tenants under Right to Buy. One remaining council tenancy property and a Right to Buy property were both then demolished. The net change in tenure by occupation is an additional five council dwellings and three homeowner dwellings. But when considering the net change by construction, the public sector added eight dwellings, despite Right to Buy and the demolition of an owner-occupied dwelling.

As data on demolitions by tenure is tracked by final occupier not by construction, this presents a problem for the methodology. Former council homes sold under Right to Buy and former private properties sold to Registered Providers will have their demolitions recorded incorrectly. The report must make an assumption about demolitions for the entire period.

Demolitions were higher in the postwar period (specifically the 1960s) and concentrated on private sector stock in slum clearances. Demolitions since 1980 were lower over a longer time period, and primarily were of social housing and private properties often built by the public sector (e.g. leaseholders on council estate regenerations).

By allocating each year's demolitions by tenure to the tenure mix of gross housebuilding for each year, we can assume that two thirds of all demolitions of properties from 1955 to 2015 were of private-built properties, and a third were of public-built properties, a reasonable estimation of the pattern described in the preceding paragraph. The individual results in each year are skewed, but the cumulative results for total underbuilding from 1955 to 2015 are satisfactory.

Denote the tenure mix of the build rate in year t for Private = θ_t^{Priv} and Public = θ_t^{Pub} respectively:

Therefore, cumulative total gross building will be:

$$H_0\left(b_1\theta_1^{Priv} + b_1\theta_1^{Pub}\right) + (H_0 + H_0(b_1\theta_1^{Priv} + b_1\theta_1^{Pub} - d_1))(b_2\theta_2^{Priv} + b_2\theta_2^{Pub}) \ldots$$

Which as $\theta_t^{Priv} + \theta_t^{Pub} = 1$ can be rewritten as:

$$H_0\left(b_1\theta_1^{Priv} + b_1\theta_1^{Pub}\right) + (H_0 + H_0(b_1 - d_1))(b_2\theta_2^{Priv} + b_2\theta_2^{Pub}) \ldots$$

And then can be split by tenure into two separate tenure specific equations:

$$H_0 b_1\theta_1^{Priv} + (H_0 + H_0(b_1 - d_1))b_2\theta_2^{Priv} \ldots + H_0 b_1\theta_1^{Pub} + (H_0 + H_0(b_1 - d_1))\, b_2\theta_2^{Pub} \ldots$$

From the identity equation, we can arrange these with the gross demolition rate to achieve the net increase in dwellings by tenure from 1955 to 2015:

In the case of the UK, from 1955 to 2015:

$$H_{Private\ Building} = 10\ \text{million} \text{ and } H_{Public\ Building} = 5.5\ \text{million}$$

$$H_{Private\ Demolitions} = 1.9\ \text{million} \text{ and } H_{Public\ Demolitions} = 1.3\ million$$

Therefore, we can calculate the change in each tenure over the period:

$$H_{Private\ Building} - H_{Private\ Demolitions} = 8.1\ million$$

$$H_{Public\ Building} - H_{Public\ Demolitions} = 4.2 \text{ million}$$

This total of private and public additions in the UK from 1955 to 2015 therefore equals:

$$H_{Change} = 12.3 \text{ million.}$$

Matching the results in Stages 2 and 3.

Part II – Calculating counterfactual values for British housebuilding

Having calculated the required figures for Britain we now move to the method used to make credible adjustments made for other countries.

The central estimate we are making is **how many extra houses the UK would have today if it had added homes at the rate required to reach the same number of homes per person as a Western European country in 2015.** As accurate values for the net change in housing stocks both in general and by tenure are not available this is calculated using each country's gross additions and UK demolitions.

However simply repeating the British method for another country for calculating how many extra homes we would have if we had built and demolished housing at the same rate of other countries will not produce a credible result for their stock of homes per person in 2015. This is because:

Other countries have different population growth rates compared to Britain. If the population growth rate is different than to the UK then the same gross increase will not equal the same change the number of homes per person. in homes per person. As all countries except Austria and Belgium had higher population growth than the UK, adjusting for this makes the average estimate more conservative.

Other countries began the period with different per person stock of homes compared to Britain. The average western European country began the period with a smaller stock of homes per person than the UK, which means some of their housebuilding after 1955 will have been to catch up to a housing stock that the UK had already built before 1955. Adjusting for this also makes the estimate more conservative.

Therefore, these effects need to be controlled for when calculating the counterfactual value of the United Kingdom so that it has:

- The same initial housing stock to population ratio as the counterfactual country.
- The same population growth rate as the counterfactual country.

Methodology

Recall the equation for changes in the number of homes per person:

$$\frac{Previous\ Housing\ Stock + Net\ Additions}{Change\ in\ Population}$$

With:

$$Annual\ Net\ Additions = Annual\ Houses\ Built - Annual\ Houses\ Demolished$$

Therefore, to estimate how many homes a counterfactual version of the UK that had adopted the housebuilding policy approach of another country (e.g. the Netherlands) in every year from 1955 to 2015 would have, the following adjustments need to be made to the British formula in Part 1 to generate an estimate of the change in the ratio of homes per person for each counterfactual version of Britain:

1. The initial stock of homes in 1955, based upon the ratio of homes per person of the comparator country
2. The rate of population growth for each year between 1955 and 2015, based on the difference in growth rates between the 'real-life' UK and the counterfactual country
3. The British rate of demolitions in every year from 1955 to 2015, adjusted to the difference in the ratios of homes per person between the 'real-life' UK and the comparator country.

We then use the counterfactual's annual housing stock to perform the same tenure calculation as the British estimate in Part 1, namely:

4. Obtaining the additional number of homes implied if Britain had had **both** the same initial per person housing stock of a European country in 1955 and had increased stock of homes per person by the same value from 1955 to 2015. This value is also subdivided by tenure.
5. Adjusting for the effects of differing changes in population each year between the European country and Britain.

Then this allows the computation of the final value for the counterfactual, using the same method in Part 1. This is comprised of the counterfactual UK's total additions to its initial adjusted housing stock. This allows us to calculate the estimate of how many homes the UK would have it had added homes at the rate of the comparator country the counterfactual is based on with:

6. The difference between the estimated final housing stock values of the counterfactual country and Britain. This is then broken down by tenure.

Stage 1: Adjust the counterfactual's initial stock of homes per person

The value for the British housing stock in 1955 is taken and then **multiplied by the ratio of stock of homes per person between the UK and the counterfactual country in 1955.**

Shown mathematically, designate the housing stock per person of Britain at time

t with a housing stock H_t and population P_t as (PC_t) and the housing stock per person of the comparator country with housing stock J_t and Population Q_t as (QK_t), such that:

$$\mathrm{PC_t} = \frac{\mathrm{H_t}}{\mathrm{P_t}} \text{ and } QK_t = \frac{J_t}{Q_t}$$

Which leads to the initial adjustment to the counterfactual's housing stock which is denoted σ_0:

$$\sigma_0 = H_0 \frac{QK_0}{\mathrm{PC_0}}$$

For our example country we will use the Netherlands. In 1955 the Netherlands had 234 dwellings per 1000 people while the UK had 302 dwellings per 1000 people.

Recall that Britain's housing stock in 1955 was $15.4\ million$. So, the adjusted initial housing stock value for the counterfactual UK based upon the Netherlands becomes:

$$\sigma_{Neth} = 15.4 \text{ million} \times \frac{234}{302} = 12\ million$$

Stage 2: Calculate net change in stock for the counterfactual UK

The overall adjusted counterfactual format for another European country takes this country's yearly gross build rate c_t and calculates the raw number of additions to the UK by multiplying this rate by a pre-adjusted housing stock.

Denote c_t is the counterfactual country's building rate and D_t is the demolition rate in year t. The exact value of the demolition rate is discussed in more detail in stage 4. The resulting formula that calculates the net change in stock in the first year before adjustments is:

$$\sigma_0 + \sigma_0 (c_t - D_t)$$

Stage 3: Adjust net change in stock for population growth

Population growth differs between countries every year. This means that an identical increase in the overall housing stock will not result in an identical increase in the stock of homes per person. Places with lower population growth do not need to build as many homes to achieve the same improvements to housing outcomes.

As Britain's population growth is lower than most of Europe in this period then this will exaggerate the difference in homes added in the counterfactual if not controlled for. Therefore, the change in population relative to Britain is updated every year and applied to the housing stock.

Denote the change in Britain's population:

$$\Delta P_t = 1 + \frac{P_t - P_{t-1}}{P_{t-1}}$$

And the change in a European country's population:

$$\Delta Q_t = 1 + \frac{Q_t - Q_{t-1}}{Q_{t-1}}$$

For the first year's net change in stock, the population growth adjustment is therefore:

$$(\sigma_0 + \sigma_0 (c_1 - D_1)) \times \frac{\Delta P_1}{\Delta Q_1}$$

The Netherland's counterfactual experiences a significantly higher rate of population growth than the UK in this period, for example the value of $\frac{\Delta P_1}{\Delta Q_1} = \frac{1}{1.02}$. Therefore, in every year the number of houses built in the counterfactual UK based upon the Netherlands is decreased according to this formula.

Also note that in the British case $\frac{\Delta P_t}{\Delta Q_t}$ equals 1 by definition and do not alter the value of the British counterfactual base used as the comparison.

Stage 4: Adjust demolition rates for population growth

Accurate and consistent demolition data does not exist for European countries apart from Switzerland.

For all other countries, the demolition data is assumed through a two-step process.

Initially, given that this is a model of Britain's housing the demolition rates are assumed to be identical to Britain's, as it is mostly pre-1947 British stock which is being demolished. Therefore, the value produced in the first round of adjusted estimates is as below, and denoted ξ_1.

$$\xi_1 = (\sigma_0 + \sigma_0(c_1 - d_1)) \times \frac{\Delta P_1}{\Delta Q_1}$$

However, the UK's demolition rate needs to be adjusted for two reasons. First, the UK's demolitions are concentrated in the 1960s due to a policy choice, with little demolition earlier or after that. Applying this to the counterfactual version of the UK building at another rate will have an inconsistent effect on the net change in dwellings.

Second, as the main target of demolitions was unfit pre-modern housing, then it is reasonable to assume that demolition rates would initially be in proportion to the stock of unfit premodern housing. If the UK had had less housing than it actually

did in 1955, then it likely would have demolished fewer dwellings in the years after.

Therefore, a value from the ratio of homes per person relative to Britain is added or subtracted from the counterfactual's demolition rate. Counterfactuals based on countries with a "head-start" in their ratio over Britain see higher initial demolitions, while counterfactuals with less housing see less.

In the case of the Netherlands, its stock of homes per person is 23 per cent lower than Britain's in 1955. Therefore, 0.23 percentage points is subtracted from the British demolition rate to calculate the counterfactual's demolition rate for that year. As Britain's demolition rate was 0.36 per cent in 1955 this makes the counterfactual demolition rate 0.13 per cent for the year.

The effect of this over time is linear and smooths out demolitions from 1955 to 2015. As the actual UK has an above-average ratio of homes per person over most countries in 1955 and is below-average by 2015, most counterfactual versions of the UK have fewer demolitions in the beginning of the period (and the 1960s) when they have fewer homes per person than the UK, and more demolitions towards the end of the period than the actual UK when most overtake the UK in terms of homes per person, and actual UK demolitions dropped to very low levels.

The net result is conservative and reduces the total amount of additional housebuilding in the European average for the counterfactual from 4.9 million to 4.3 million.

Denote ν_t the value of the counterfactual housing stock in year *t*. Therefore in 1955 at the end of year 1 the housing stock (ν_1) is:

$$\nu_1 = \left(\sigma_0 + \sigma_0 \times \left(c_1 - \left(d_1 - \left(1 - \frac{\xi_1}{PC_1}\right)\right)\right)\right) \times \frac{\Delta P_1}{\Delta Q_1}$$

Denote $\left(d_t - \left(1 - \frac{\xi_t}{PC_t}\right)\right)$ as D_t for all following expressions.

An exception is made for Switzerland, where the country's demolition data is of high enough quality to be directly inputted into the model.

Stage 5: Calculate and adjust the counterfactual additions:

Therefore, given all the above these, Part 1 is applied to the counterfactual values established above as follows. Total Gross Building for the counterfactual version of the UK for each country will be:

$$J_{Total\ Building} = \sum c_1\nu_1 + c_2\nu_2 \ldots$$

For the Netherlands counterfactual: $J_{Total\ Building} = 23\ million$

And Total Demolitions will be:

$$J_{Total\ Demolitions} = \sum D_1 v_1 + D_2 v_2 \ldots$$

For the Netherlands counterfactual: $J_{Total\ Demolitions} = 1.7\ million$

Therefore, total net additions will be:

$$J_{Net\ Building} = J_{Total\ Building} - J_{Total\ Demolitions}$$

For the Netherlands counterfactual:

$$J_{Net\ Building} = 23\ million - 1.7\ million = 21.3\ million$$

These values are then broken down by tenure in exactly the same method as Part I.

So total additions by tenure are:

Net Private Additions:

$$J_{Net\ Private\ Building} = J_{Total\ Private\ Building} - J_{Total\ Private\ Demolitions}$$

For the Netherlands counterfactual:

$$J_{Net\ Private\ Building} = 13.6\ million - 1\ million = 12.6\ million$$

Net Public Additions:

$$J_{Net\ Public\ Building} = J_{Total\ Public\ Building} - J_{Total\ Public\ Demolitions}$$

For the Netherlands counterfactual:

$$J_{Net\ Public\ Building} = 9.4\ million - 0.7\ million = 8.7\ million$$

Stage 6: Adjust for population effects on gross building and gross demolitions

In Stage 3 we saw that the impact of population growth upon the net change in stock is controlled for by multiplying the change in population ratios between the UK and the comparator country. But we have not yet adjusted annual gross building and gross demolitions for population growth, which are used to calculate the net change by tenure. The sum of gross building and gross demolitions should equal net change in stock.

The changes in the housing stock between periods one and two is currently:

$$\nu_2 - \nu_1 = (\nu_1 + \nu_1(c_2 - D_2))\frac{\Delta P_2}{\Delta Q_2} - \nu_1$$

However the sum of gross housebuilding and gross demolitions between period one and two will be:

$$v_1(c_2 - D_2)$$

Therefore, there is a discrepancy between the total net change in stock and the sum of gross building and gross demolitions due to the population adjustments. Due to compounding, this discrepancy runs into millions more homes in the counterfactual from 1955 to 2015 – adjusting for this makes the estimate more conservative.

Denote the discrepancy as γ_t

$$\gamma_1 = \nu_1\left(1 - \frac{\Delta P_2}{\Delta Q_2}\right)(c_2 - D_2 + 1)$$

Therefore, the sum of these discrepancies will be:

$$J_{Population\ Adjustments} = \sum \gamma_1 + \gamma_2 \dots$$

Therefore, the total net additions into a country's housing stock will be:

$$J_{Total\ Additions} = J_{Net\ Building} - J_{Population\ Adjustments}$$

For the Netherlands counterfactual this is:

$$18.6\ \text{million} = 21.3\ \text{million} - 2.7\ \text{million}$$

We then assume that these adjustments occur in the same tenure ratio as the total tenure ratio of the net additions to the housing stock.

We are now able to calculate the number additions to the housing stock implied by the counterfactual:

Therefore, for Private Additions:

$$J_{Private\ Additions} = J_{Net\ Private\ Building} - \frac{J_{Private\ Building}}{J_{Total\ Building}} \times J_{Population\ Adjustments}$$

For the Netherlands counterfactual this is:

$$11\ million = 12.6\ million - 0.59 \times 2.7\ million$$

Public Additions

$$J_{Public\ Additions} = J_{Net\ Public\ Building} - \frac{J_{Public\ Building}}{J_{Total\ Building}} \times J_{Population\ Adjustments}$$

$$7.6\ million\ =\ 8.7\ million\ -\ 0.41\ \times 2.7\ million$$

Stage 7: Adjust for the initial differences in the stock of homes per person and compare to the UK.

The aim of the counterfactuals was to produce **how many extra houses Britain would have of each tenure if we had added homes at the net rate to reach the estimated current per capita housing stock of another European country.**

This is simply the value obtained by taking the difference between the final (2015) values of the stock of homes per person of Britain (H_{Final}) and the counterfactual country (J_{Final}), as the estimate for the counterfactual has already adjusted for differences in population growth, demolitions, and importantly, the initial difference in homes per person in 1955 in the steps discussed above.

$$J_{Final} - H_{Final} = Total\ Surplus$$

In the case of the Netherlands counterfactual: $J_{Final}\ =\ 30.6\ million$, Therefore:

$$Total\ Surplus\ =\ 30.6\ million - 27.7\ million\ =\ 2.9\ million$$

However, the total surplus is not simply the value that Britain would have if we had increased the stock of homes per person by the same gross amount as the counterfactual country:

$$J_{Total\ Additions} - H_{Total\ Additions} \neq Total\ Surplus$$

This is because we are controlling for convergence so we need to take into account the initial deficit or surplus.

Recall that σ_0 refers to the initial 1955 housing stock of the counterfactual, adjusted for the differences in the ratio of homes per person between actual UK and the comparator country; and that H_0 refers to the initial 1955 housing stock of the UK.

Therefore:

$$J_{Final} = (\sigma_0\ + J_{Total\ Additions})$$

$$H_{Final} = (H_0\ +\ H_{Total\ Additions})$$

This means the final estimate of the difference in housebuilding between the UK and any counterfactual is:

$$(\sigma_0 + J_{Total\ Additions}) - (H_0 + H_{Total\ Additions}) = Total\ Surplus$$

In the case of the Netherlands: $\sigma_0 = 12\ million$ and $J_{Total\ Additions} = 18.6\ million$ Therefore:

$$J_{Final} = 12\ million + 18.6\ million = 30.6\ million$$

In the case of the United Kingdom: $H_0 = 15.4\ million$ and $H_{Total\ Additions} = 12.3\ million$.

Therefore:

$$H_{Final} = 15.4\ million + 12.3\ million = 27.7\ million$$

$$Total\ Surplus = 30.6\ million - 27.7\ million = 2.9\ million$$

Notice how the initial difference in housing stock implied by the different ratios of housing per person are controlled for in this calculation. The $2.9\ million$ figure does not include the housebuilding required for the Netherlands to catch up with the UK.

Unless this initial gap in homes per person is controlled for by tenure then the difference between the private and public surpluses will not equal the total lack of housing. Therefore, to control by tenure the equation can be rewritten as:

$$\sigma_0 - H_0 + J_{Total\ Additions} - H_{Total\ Additions} = Total\ Surplus$$

Where $\sigma_0 - H_0 = J_{Diff}$ and is equivalent to the initial difference in housing stock sizes = $12\ million - 15.4\ million = -3.4\ million$

Therefore, as with the cumulative population effects, it is assumed that the initial gap was made up by the same tenure ratio as the total build of the counterfactual country:

For Private Values:

$$\frac{J_{Private\ Building}}{J_{Total\ Building}} \times J_{Diff} + J_{Private\ Additions} - H_{Private\ Additions} = Private\ Surplus$$

For the Netherlands this is:

$$0.59 \times -3.4\ million + 11\ million - 8.1\ million = 0.9\ million$$

For Public Values:

$$\frac{J_{Public\ Building}}{J_{Total\ Building}} \times J_{Diff} + J_{Public\ Additions} - H_{Public\ Additions} = Public\ Surplus$$

For the Netherlands this is:

$$0.41 \times -3.4\ million + 7.6\ million - 4.2\ million = 2\ million$$

Creating the values seen in the table.

Additional Notes

It is assumed that:

$$\frac{J_{Public\ Building}}{J_{Total\ Building}} \times J_{Diff} + J_{Public\ Additions} > 0$$

Which translates to the assumption that, adjusting for population growth, under no circumstances would the UK have demolished more social housing from 1955 to 2015 than has been built. This controls for a limitation for the model in its handling of the population discrepancy in Stage 6 with countries that build very little social housing – the UK counterfactuals of Switzerland and Belgium without this assumption experience net losses of social housing of 421,000 and 677,000 social homes respectively from 1955 to 2015, even though neither actual Switzerland nor actual Belgium appear to have demolished social housing at that scale.

Finally, any discrepancies between the example values in the methodology section and the table in the report is due to the impact of rounding at each stage in the methodology section, which is not in the data processing in the counterfactual for the report, where rounding is only done for the final values.

Interpolation

In the United Nations data, housing stock values are not given consistently across Western Europe before 1960 and there are occasional gaps of a year or two for certain countries. To overcome this problem and create consistent housing stock values for the purposes of creating housebuilding and demolition rates two linear interpolation methods are used.

When there is a gap **between** years in reported housing stocks, the total net additions (or gross, if reported demolitions are not available) between the two known values are taken. The surplus or deficit between these values is then divided by the number of years between the two values and the number is then added and subtracted to the housing stock for each year. For example, the given census value for Austria is 2.14 million homes in 1951 and then not reported until 1963 when it is given as 2.36 million homes. During this twelve years gap, 490,000 homes were built, which implies 270,000 demolitions. We therefore assume 270,000 / 12 demolitions were carried out each year.

For historic England and Welsh estimates, a more detailed interpolation is possible, where the demolitions between the total difference in housing stock between two dates are assumed to be distributed **proportionally** to the gross building in each year. For example, in England are Wales there are 3.9 million houses in 1861 and 4.5 million in 1871. In this case as 750,000 houses were built in this period this implies that 150,000 demolitions were made. Therefore, the value of demolitions every year is calculated by:

$$(Annual\ Gross\ Building / 750{,}000) \times 150{,}000$$

When there is no data **before** a given year reported housing stock value then the **ratio** between the values of the number of demolitions is taken from the first known year and assumed to hold for all previous years. This ratio is then used to calculate an estimated number of demolitions for each year through multiplying the total gross builds for each unknown year with this ratio. After demolitions are obtained, then the net additions for each year is found and subtracted from the previous value for housing stock.

The only country this applies to from before 1955 is Belgium, which gives a housing stock value of 3.2 million in 1963. In 1962 46,000 homes were built and 2,000 were demolished. Therefore, a ratio of 2:46 was assumed for demolitions to construction was assumed for all previous years. Between 1956 and 1963 325,000 houses were built, implying 14,000 total demolitions. This gives a value of 310,000 net additions from 1955 onwards, which implies a housing stock of approximately 2.9 million in 1955.

For graphs methodological changes mean that discontinuities can arise – for instance, some countries' data appears to alternate between including and dropping vacant dwellings. For example, after the UN data source ends in 2000, many countries switch to counting inhabited dwellings only. In certain cases, the statistical authority will provide inhabited data before 2000 as well, and this data,

although not being used in earlier calculations, is used in the graphs to avoid a discontinuity. If this is not possible then a consistent interpolated estimate of the housing stock using the first method is used for graphs.

For all graphs and estimates of the net change in the stock of homes per person a methodologically consistent form has been used in each case.

For tenure, the ratio of private sector to public sector construction is taken and, if the gap is four years or less then undergoes linear interpolation. Otherwise, the final or initial ratio of tenure is assumed to hold for all previous or subsequent years. The recorded value for construction is then multiplied by this ratio and therefore assumed to be construction by tenure. However, in practice large gaps in specification by tenure mainly occur for countries that build minimal social housing.

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